March 18, 2014
Hiroshi Michiwaki

Division by Zero

Proof of 0/0 = 0 (subtractable set theory)

Definition: When A and B are non-negative real numbers, calculation of division B/A is defined as follows:

$$|B| - (|\{A\}| \cdot |A| + a) = 0 \ (0 \leq a)(A, B, a \in \mathrm{R})$$

where $B_0 = B$, $A_j = A$ ($j = 0, 1, 2, \ldots$) The set based on A_{j+1} (this A_{j+1} is called the j+1[th] subtractable number) in the subtractable recurrence relation $B_j - A_{j+1} = B_{j+1}$ is defined as the j+1[th] subtractable number set $\{A_{j+1}\}$, and when B_j (this B_j is called the j[th] subtracted number) satisfies $B_j > B_{j+1} \geq 0$, $\{A_{j+1}\} \neq \emptyset$, and when it does not satisfy this condition, $\{A_{j+1}\} = \emptyset$. The set whose elements are all within the subtractable number set $\{A_{j+1}\}$ is defined as a subtractable set $\{A\}$.
Here, B is the dividend, A is the divisor, and $|\{A\}|$ is the number of elements in the subtractable set $\{A\}$, which is the quotient of B/A, where a is the remainder, and also the minimum non-negative real number when $|\{A\}|$ is maximized.

At this time, the following holds:

Theorem 1: $0/0 = 0$.
Proof: Based on the definition $|B| - (|\{A\}| \cdot |A| + a) = 0$, $B = 0 \Rightarrow a = 0$. Therefore,

$$0 - (|\{A\}| \cdot |A| + 0) = 0 - |\{A\}| \cdot |A| = 0$$

where $A = 0 \Rightarrow 0 - |\{A\}| \cdot |A| = 0 - |\{A\}| \times 0 = 0$.

Now, because $B = B_0 = 0 \wedge A = A_j = 0$, the subtractable recurrence relation $B_j - A_{j+1} = B_{j+1}$ is $0 - 0 = 0$, and $B_j > B_{j+1} \geq 0$ is not satisfied. Therefore, the subtractable set $\{A\} = \{\{A_1\},\{A_2\},\ldots,\{Aj\},\ldots\} = \{\emptyset,\emptyset,\ldots,\emptyset,\ldots\} = \emptyset$, and the number of elements $|\{A\}| = |\emptyset| = 0$ is obtained. Therefore,

$$A = B = 0 \Rightarrow B - (|\{A\}| \cdot |A| + a) = 0 - (0 \times 0 + 0) = 0$$

$\therefore 0/0 = 0$.

Proof of 100/0 = 0 (subtractable set theory)

Theorem 2: The quotient of division B/A when $A = 0$ is 0, and the remainder is B.
Proof: Based on the definition $|B| - (|\{A\}| \cdot |A| + a) = 0$, if $A = 0$,

$$B = |\{A\}| \cdot |A| + a = |\emptyset| \cdot |0| + a = 0 \times 0 + a = a$$

If both sides are divided by $A = 0$,

$$\frac{B}{A} = \frac{|\{A\}| \cdot |A| + a}{A} = \frac{|\{A\}| \cdot |A|}{A} + \frac{a}{A} = \frac{|\emptyset| \times |0|}{0} + \frac{a}{0} = \frac{0 \times 0}{0} + \frac{a}{0} = \frac{0}{0} + \frac{a}{0} = 0 + \frac{a}{0} = \frac{a}{0}$$

is obtained. However, $0/0 = 0$ is based on Theorem 1. Here, $B \neq 0 \Rightarrow a \neq 0$, (left side) = B/A = $B/0$ (right side) = $(a/a)/(0/a) = 1/0$.

Therefore,

$$\frac{B}{0} - \frac{1}{0} = 0$$

$$\frac{B-1}{0} = 0$$

are obtained. In other words, $a/0 = 0$ is established. Of course, a is the minimum non-negative number when $|\{A\}|$ is maximized; thus, the quotient of division B/A when $B \neq 0$ and $A = 0$ is 0, and the remainder is B.

This means that the extended definition of division by the subtractable set states that in division by zero, even if the quotient also becomes 0, the information of the remainder term is strictly retained.

Proof of 100/0 = 0 (general fraction theory)

Theorem 2: $x/0 = 0 (x \in \mathrm{R})$.
Proof: Assume $F(B,A) = B/A$. At this time, if a $\neq 0$,

$$F(a, 0) = F(a/a\ , 0/a) = F(1,0)$$

$\therefore F(a, 0) = F(1,0)$.

On the other hand,

$$F(a, 0) = F(a \times 1,0) = aF(1,0)$$

Therefore,

$$aF(1,0) = F(1,0)$$

and

$$(a-1)\,F(1,0) = 0$$

$\therefore F(1,0) = 0/(a-1)$ is obtained.

Here, $a \neq 1 \Rightarrow F(a,0) = F(1,0) = 0$ is established. If $a = 1$,

$$F(1,0) = \frac{0}{1-1} = \frac{0}{0} = 0$$

is obtained.

Proof of $x/0 = 0$ (special fraction function)

Definition: Division B/A is defined by $AB/A^2 (A, B \in \mathrm{R})$. At this time, the next theorem is established.

Theorem 3: $x/0 = 0 (x \in \mathrm{R})$.
Proof: Based on the definition,

$$A = 0 \Rightarrow \forall B\{AB/A^2 = (0 \times B)/0^2 = (0 \times B)/(0 \times 0) = (0 \times B)/0 = B \times 0/0 = B \times 0 = 0\}$$

$\therefore B/0 = 0$.

Supplement 1: In the definition of division B/A, AB/A^2, note that $A \neq 0 \Rightarrow AB/A^2 = B/A \wedge A = 0 \Rightarrow A/A \neq 1$.

Supplement 2: Note that the expansion of the equation $B \times 0/0 = B \times 0$ uses $0/0 = 0$.

Lemma: In the hyperbolic function $f(x) = a/x (0 < a)$, if the domain is $-\infty < x < +\infty$, the range becomes $-\infty < y < +\infty$, but in the area sufficiently close to $x = 0$; in other words, $x \to \pm\varepsilon$ ($0 < \varepsilon < E$: a constant that is smaller than a sufficiently small arbitrary constant),

$$\lim_{x\to+\varepsilon} f(x) = \lim_{x\to+\varepsilon} \frac{ax}{x^2} = \lim_{x\to+\varepsilon} \frac{a}{x} = +\infty$$

Also,

$$\lim_{x\to-\varepsilon} f(x) = \lim_{x\to-\varepsilon} \frac{ax}{x^2} = \lim_{x\to-\varepsilon} \frac{a}{x} = -\infty$$

Of course this notation means

$$\lim_{x\to+\varepsilon} f(x) = \lim_{x\to+0} f(x)$$

($0 < \varepsilon < E$: a constant that is smaller than a sufficiently small arbitrary constant).

At the first glance, it appears as if $f(x)$ is $-\infty = +\infty$ at $x = 0$, but according to the concept of limits, $x \to +0$ means x approaches 0, but never reaches 0; x does not leave the positive side of 0. Therefore, clearly,

$$\lim_{x\to+0} f(x) \neq \lim_{x\to-0} f(x) \wedge -0 < +0$$

is established. On the other hand, if $x = 0$ in $f(x)$ (such operation is called localization),

$$\operatorname*{loc}_{x=0} f(x) = \operatorname*{loc}_{x=0} \frac{a}{x} = \frac{a}{0} = 0$$

is established. This means that around ± 0 near 0, those functions with divergence approaching opposite poles $\pm\infty$ become $f(0) = 0$ at the center point. Therefore, the problem of $-\infty = +\infty$ is solved. Therefore; it becomes:

$$\lim_{x\to -0}\frac{a}{x} = -\infty < \frac{a}{0} = 0 < \lim_{x\to +0}\frac{a}{x} = +\infty$$

It can be said that limits and division by zero are different concepts. On the other hand, note that the concept and operation of limits are not division by zero.

Discussion concerning rotational systems

Definition: Tangential velocity v at rotational radius r in a rotation system that rotates at an angular velocity of ω is used to establish $v = r\omega$. At this time, the following theorems are established:

Theorem 4: The rotational center of a rotational system does not rotate.
Proof: If we imagine a right-angle triangle with a base of r and a height of v, its slope $\omega = v/r > 0$ becomes $\omega = 0$ at $r = 0$ based on Theorem 2. If we substitute physical values for rotational radius, tangential velocity, and angular velocity for r, v, and ω, respectively, the result shows that the rotational center does not rotate.

Theorem 5: A point does not rotate.
Proof: If we assume that point P is rotating, point P has an angular velocity of $\omega > 0$. Therefore, it has a tangential velocity of $v > 0$. At this point, because $r = v/\omega > 0$, point P satisfies the radius $r > 0$. However, the assumption says that the point has $r = 0$, which is inconsistent. Such inconsistency is caused by the assumption that the angular velocity of point P is $\omega > 0$. Therefore, point P must have an angular velocity of $\omega = 0$.

The relationship of $\omega = v/r$, $x/0 = 0$, and differential science

If we use the tangential velocity v at the rotational radius r of a rotational system that rotates with an angular velocity of ω, it can be expressed as $v(r) = \omega r$. If we differentiate v for r:

$$\lim_{\Delta r\to\varepsilon}\frac{\omega(r+\Delta r)-\omega r}{\Delta r} = \lim_{\Delta r\to\varepsilon}\frac{\omega\Delta r}{\Delta r} = \lim_{\Delta r\to\varepsilon}\frac{\Delta r}{\Delta r}\omega = \omega$$

However, it is:

$$0 < \varepsilon < E$$

and E is a constant smaller than any small constant.

What is important here is that ε must be:

$$0 < \varepsilon \wedge \varepsilon \neq 0$$

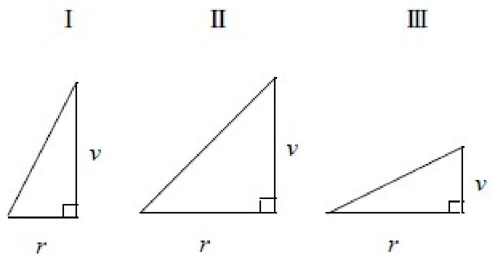
This is the true nature of differentiation. ε infinitely approaches 0, but is never equal to 0. Otherwise, εbecomes $\Delta r \to 0$, and

$$\lim_{\Delta r\to 0}\frac{\omega(r+\Delta r)-\omega r}{\Delta r} = \lim_{\Delta r\to 0}\frac{\omega\Delta r}{\Delta r} = \lim_{\Delta r\to 0}\frac{\Delta r}{\Delta r}\omega = \lim_{\Delta r\to 0}\frac{0}{0}\omega$$

This would mean that differential science implicitly assumes 0/0 = 1.

In other words, differentiation is established by $\Delta r \to \varepsilon$ ($\varepsilon \neq 0 \wedge 0 < \varepsilon < E$: E is a constant smaller than any small constant), and cannot reach 0. In differentiation, Δr can approach a point but cannot reach it. This means that the operation of taking the limit in differentiation is dynamic. For example, in differential science, infinitely small values of high order, etc., cannot be handled, and extremely small ranges cannot be handled strictly.

Now let us consider the following three right-angle triangles:



In the case of triangle I, $r < v$, and if v approaches 0 in a shape similar to $\omega_{\mathrm{I}} = v/r$, it approaches 0 while maintaining the relationship $r < v$. If v reaches a length of 0; in other words, if it is localized, it becomes a point with a size of 0. At this time, the slope would be 0. Of course, because $r < v = 0$, $r = 0$.

At this time, because $\omega_{\mathrm{I}} > \omega_{\mathrm{II}} > \omega_{\mathrm{III}}$, like triangle I, triangles II and II are also localized at one point, and ultimately converge to $\omega_{\mathrm{I}} = \omega_{\mathrm{II}} = \omega_{\mathrm{III}} = 0$.

In differential science, triangle cannot reach this peak; thus, it is looking at the slope of a linear approximation line with a point that is infinitely close to the peak and the peak. This result is not inconsistent with 0/0 = 0, and is not at all inconsistent with differential science; thus, it can even be said to be complimentary.