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Straight line in a coordinate space & "Division by zero"

The equation for line C in the x-y coordinates plane, where "a" is the slope and "b" is the y-intercept, the equation is:

$$y = ax + b \qquad (1)$$

When $\mathrm{P}_1(x_1, y_1)$ and $\mathrm{P}_2(x_2, y_2)$ are two different points on line C, slope "a" is:

$$a = \frac{y_2 - y_1}{x_2 - x_1} \qquad (2)$$

and y-intercept "b" is:

$$b = y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1 = y_2 - \frac{y_2 - y_1}{x_2 - x_1} x_2 \qquad (3)$$

By applying equations (2) and (3) into equation (1), the equation for line C will be:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1, \quad y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2 \qquad (4)$$

In traditional mathematics of "Non-Division by zero", only $x_1 \neq x_2$ could be applied to equation (4), and in the condition of $x_1 = x_2$, suddenly this exceptional formula is given.

$$x = x_1 \qquad (5)$$

Here, line C becomes parallel to the y-axis, and the denominator of equation (2), which expresses slope "a", becomes 0 (Zero) and "Division by zero" occurs. It's been said that the value diverges and becomes incalculable. However, when "Division by zero" mathematics that is expanded in a natural way is applied to equation (4), point $\mathrm{P}(x_j, y_j)$ on line C when $x_1 = x_2$, under the condition $x_j = x_1$, the left equation will be:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 = \frac{y_2 - y_1}{x_1 - x_1}(x_1 - x_1) + y_1 = \frac{(y_2 - y_1) \times 0}{0} + y_1 = \frac{0}{0} + y_1 = y_1 \qquad (6)$$

and similarly, from the relationship between the two equations, we will earn:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_2 = y_2 \qquad (7)$$

In general, with any two different points $\mathrm{P}_j(x_j, y_j)$ and $\mathrm{P}_k(x_k, y_k)$ on line C, the equation of line C must be the same as that of equation (4).

$$y = \frac{y_k - y_j}{x_k - x_j}(x - x_j) + y_j, \quad y = \frac{y_k - y_j}{x_k - x_j}(x - x_k) + y_k \qquad (8)$$

When $x_j = x_k$, i.e. line C is parallel to the y-axis,

$$y = y_j \wedge y = y_k \quad (y_j \neq y_k) \qquad (9)$$

Considering that y_j and y_k are the y-coordinates of any two points: $\mathrm{P}_j(x_j, y_j)$ and $\mathrm{P}_k(x_j, y_k)$ on line C, it can be understood that y_j and y_k in equation (9) span all y-coordinates on line C. Let y_c be the set of all y-coordinates on line C, equation (9) will be:

$$y = y_{\mathrm{C}} \quad (x_j = x_k) \qquad (10)$$

Therefore, there is no contradiction for all line C in the x-y coordinate space, including lines parallel to the y-axis. It was shown that the calculation result can be obtained correctly by applying the equation of line C expressed in equation (8).

Now, when $x_1 = x_2$ and considering $y_1 \neq y_2$, by applying the method of "Division by zero" to equation (2), the equation for slope "a" will be:

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{x_1 - x_1} = \frac{y_2 - y_1}{0} = (y_2 - y_1) \times 0 = 0 \qquad (11)$$

This helps to understand that slope "a" of line C parallel to the y-axis is 0 (Zero). On the other hand, considering that slope "a" is 0 and $x_1 = x_2$, applying the method of "Division by zero" to equation (3) in order to find the y-intercept value, the equation will be:

$$b = y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1 = y_1 - \frac{y_2 - y_1}{x_1 - x_1} x_1 = y_1 - \frac{y_2 - y_1}{0} x_1 = y_1 - (y_2 - y_1) x_1 \times 0 = y_1 \qquad (12)$$

Considering the relationship between equation (8) and (10), equation (12) will be:

$$b = y_{\mathrm{C}} \quad (x_j = x_k) \qquad (13)$$

However, y_{C} is the set of all y-coordinates on line C.
Consequently, when $x_j \neq 0$, this results in the possible existence of a y-intercept despite being off the y-axis. At first glance, this seems like a contradiction. However, what gives this feeling of contradiction is not the essence. The reason is, in the equation of the line given by equation (1), the point where the y-coordinate crosses the y-axis when $x = 0$ is named the y-intercept. Since we assume $x_j \neq 0$ in the first place, giving the condition $x = 0$ is clearly inconsistent. The concept of the y-intercept is that it does not necessarily include the existence of a line where the line C does not always intersect with the y-axis. Therefore, there is no concept of a y-intercept in a line C ($x \neq 0$) parallel to the y-axis, and it is clear that "b" merely represents the y-coordinates of line C.

From the above, it is clearly shown that even if the method of "Division by zero" is applied to the line equation, it does not cause any contradiction. This result rather improves the discontinuous relationships such as the relationship of equation (5) that suddenly appears in the traditional mathematics of "Non-Division by zero", and also shows an extremely beautiful expansion that one unified equation can express all cases correctly.